Title: Homecoming Tickets

Link to Outcomes:

- **Problem Solving** Students will solve a practical problem through logic and mathematics.

- **Communication** Students will translate verbal descriptions of physical conditions into algebra.

- **Reasoning** Students will observe and generalize the validity of several possible answers.

- **Algebra** Students will graph inequalities, find graph intersections, and solve systems of equations.

- **Technology** Students will use a computer and graphing software, or a graphing calculator, as a tool in a decision-making process.

Brief Overview:

Students will use principles of linear programming to solve a multi-condition problem. By successively eliminating possible solutions, they will derive a range of usable solutions.

Grade/Level:

Grades 9-12, Algebra/Algebra 2

Duration:

Two days. Pre-Lab in class, Lab activity in Computer Lab

Prerequisite Knowledge:

Students should be able to graph inequalities reasonably well. Although graphing software will be used as a computational aid, students must be able to interpret the results.

Objectives:

Students will**:**

- translate English expressions into algebraic form.
- create equations and inequalities.
- solve equations and inequalities for a given variable.
- graph equations and inequalities.
- find intersection points by inspection.
- solve for intersection points using algebra.
- apply results to the solution of the problem.

Materials/Resources/Printed Materials:

- Computer and graphing software or graphing calculator
- Student Worksheet - Student Resources
- Graph paper
- Student Worksheet Answer Key -Teacher Resource #1
- Teacher's Guide for Using Derive as a Graphing Tool - Teacher Resource #2

Development/Procedures:

Teacher will arrange students in groups according to available equipment, i.e. the number of computers or graphing calculators available. Groups of 2 or 3 might be appropriate. Each student will complete his own answer sheet, relying on peers for verification, and for assistance on the computer/calculator.

Activity 1: The Equation — Consists of the statement of the problem and the restricting condition. The students should be able to complete the equations with minimal teacher guidance. This activity should be done as a pre-lab activity.

Activity 2: The Graph — Should be done with computers or graphing calculators. The emphasis is on the conclusion rather than the process - using the computer as a tool. The teacher should lead the group in setting up the program and entering the equations. It is important that the students also draw the graphs on their paper. Notice the reversal of the shading of the inequality, since we are eliminating, rather than including possibilities. The teacher must take time to ensure that all students have properly constructed their graphs. Finding the intersections will be difficult and can be accomplished by group consensus.

Activity 3: Conclusion — Could be a post-lab activity. If time allows, it could also be done immediately, bringing closure to the activity. Some students will have trouble with the idea that there is not one 'correct' answer.

Evaluation:

It is important that students are checking their work at several stages: Equations will be checked upon completion; graphs will be checked by inspection; and conclusions will be verified in groups.

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HOMECOMING TICKETS

We are busy making plans for our homecoming game. Since our team has been doing quite well, and because our facilities are very small, we will be faced with the unusual dilemma of limiting our ticket sales. We will use a technique called linear programming to determine the maximum number of adult tickets and student tickets that we can sell.

Assumptions

1. There are only 200 seats in our stadium, so the sum of adult tickets and student tickets must be less than or equal to 200.
2. For security purposes, we want to have at least one adult for every five students.
3. The total from ticket sales must be at least $600. Student tickets are set at $3, and adult tickets are set at $5 each.
4. For spirit purposes, we want the number of students to be at least twice as much as the number of adults.

The Equations

Using these conditions, begin forming the algebraic equations:

Assign variables:
_______ => number of adults ___________=> number of students

1. The number of adult tickets plus the number of student tickets is less than or equal to 200.
   ____
2. The number of adults must be at least one-fifth the number of students.
   ____
3. The ticket income from students will be 3 times the number of students. _______ The ticket income from adults will be 5 times the number of adults. _____ The total income will be the sum of these two values. _______. This value must be greater than or equal to 600.
   ____
4. The number of students must be at least twice the number of adults.
   ____

Now solve each of these equations for whichever variable you have representing the number of adults.

1. ____
2. ____
3. ____
4. ____

Check these equations with your instructor.

Graphing

Using a graphing utility, graph each of these inequalities. Since we are interested in eliminating those values that do not fit our conditions, shade in the area that does not fit the inequality. In a sense, you are reversing the shading.

As you graph each line on the utility, copy that line to your graph paper. Be sure that you are very neat, and that you label each line!

When you have the four lines graphed and transferred to your paper, check with your instructor.

You should have created a strange little quadrilateral. Any point within this quadrilateral will satisfy all of the conditions. (By shading the areas that do not meet the qualifications, all you are left with are those that do meet the qualifications.) The corners of this quadrilateral, called the critical points, will represent the maximum and minimum points.

Conclusions

1. Find the coordinates of each of the four critical points.

2. Find the total income at each critical point. (Remember that each student ticket sells for $3, and each adult ticket sells for $5.

3. The parent volunteer that is volunteering to sell the tickets for us, is not very interested in optimal values, but rather in how many tickets she can sell. What is the maximum number of adult tickets she can sell? (Hint: The answer is not 200. Look at your critical points.)
   What is the highest adult value? _______
   What is the largest number of student tickets? _____

Teacher Resource #1

HOMECOMING TICKETS
Answers

The first set of equations:

a=> number of adults

s=> number of students

1. $a + s \le 200$
2. $a \ge 1/5s$
3. $5a + 3s \ge 600$
4. $2a \le s$

1. $a \le -s + 200$
2. $a \ge 1/5s$
3. $a \ge -3/5s + 120$
4. $a \le 1/2s$

The critical points

(students, adults)

(166.66,33.33) (150,30) (133.33,66.66) (109.09,54.54)

Income generated

(166.66,33.33)	=>	667
(150,30)	=>	600
(133.33,66.66)	=>	733
(109.09,54.54)	=>	600

Conclusion:

Maximum number of tickets that they may sell to students is 166.
Maximum number of adult tickets is 67.

TEACHER'S GUIDE FOR USING DERIVE AS A GRAPHING TOOL.

SETUP

With program properly loaded, have each student enter the following commands (they are magic, don't worry about them yet.)

Type the indicated letter for each command:

<O>ptions **<D>**isplay **<G>**raphics **<H>**igh **<L>**arge **<E>**xtended **<V>**GA **<ENTER>**

<W>indow **<S>**plit **<V>**ertical **<ENTER>** for Column 40

<F1> to move to the second window
<W>indow **<D>**esignate **<2>**d-plot

<S>cale 100 **<TAB>** 100 **<ENTER>**

see *Figure d-1*

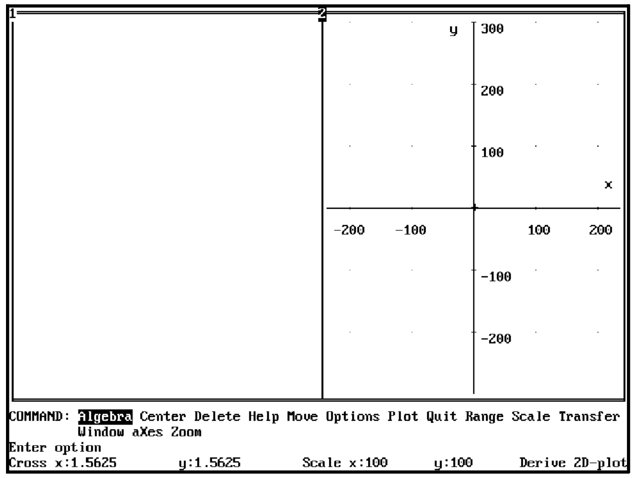

Figure d-1

FUNCTIONS

<F1> to Window 1, enter functions

<A>uthor $a+s=200$ **<ENTER>**

so**<L>**ve, **<ENTER>** for expression #1, variable a **<ENTER>**
<F1> to go to Window 2 **<P>**lot

see *Figure d-2*

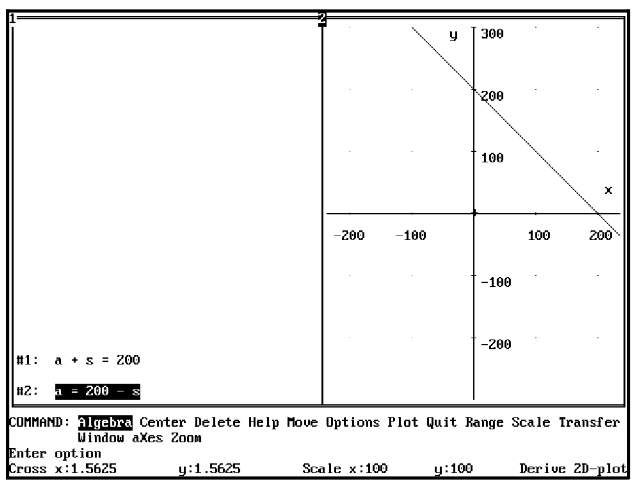

Figure d-2

<F1>
<A>uthor *a* = 1/5*s* **<ENTER>**

<F1>
<P>lot

see *Figure d-3*

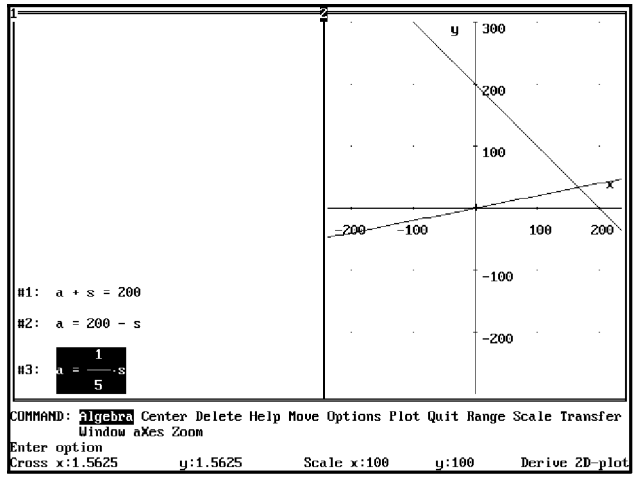

Figure d-3

<F1>
<A>uthor   3*s* + 5*a* = 600   **<ENTER>**

so**<L>**ve expression #4 **<ENTER>** variable a **<ENTER>**

<F1>
<P>lot

see *Figure d-4*

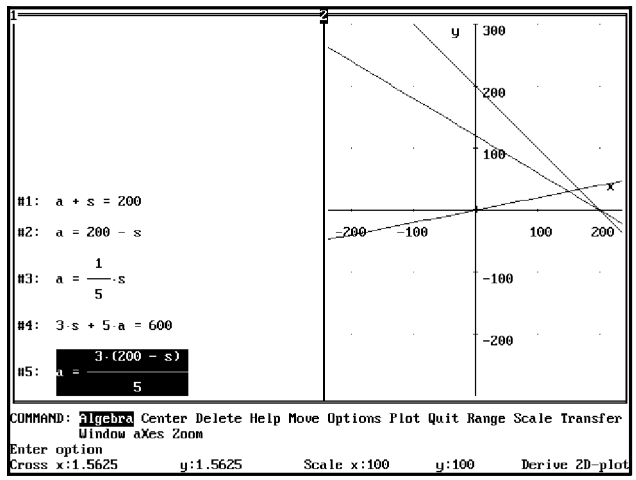

Figure d-4

<F1>
<A>uthor   2*a* = 1*s*   **<ENTER>**

so**<L>**ve expression #6 **<ENTER>** variable a **<ENTER>**

<F1>
<P>lot

see *Figure d-5*

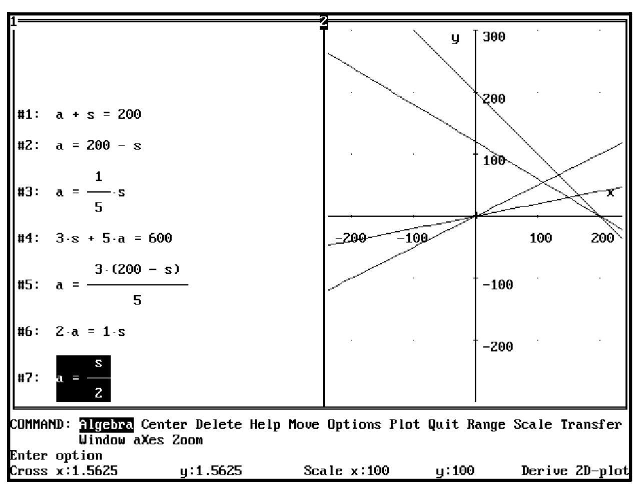

Figure d-5

FINDING CRITICAL POINTS - Points of Intersection

Make sure you are in screen #2, if not **<F1>**
Use your arrows to move the cross hairs to the center of the critical region

<C>enter **<Z>**oom **<B>**oth **<I>**n **<ENTER>**

see *Figure d-6*

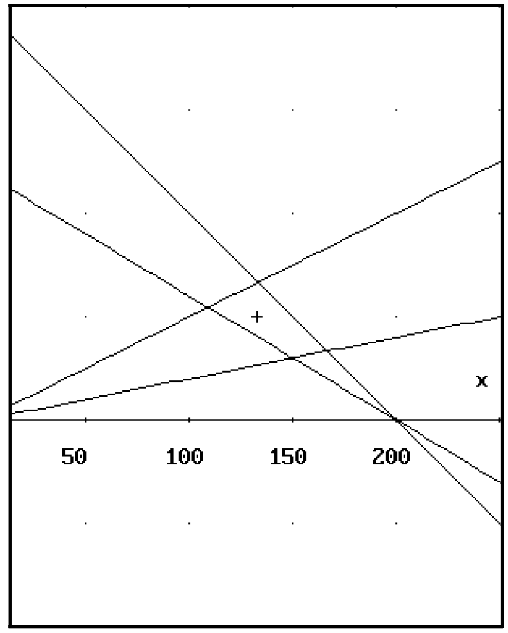

Figure d-6

Now we need to determine our critical points. Using the Trace function **<F3>**, we can trace the lines using the left and right arrows. Change lines by using the Up and Down arrows.

See *Figures d-7* through *d-10*.

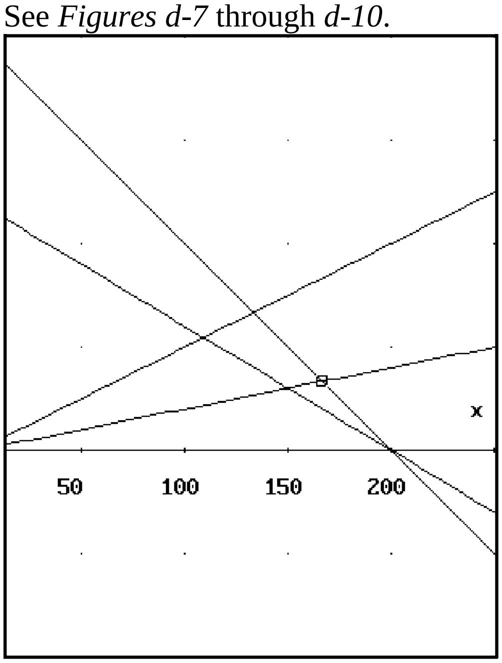

Figure d-7

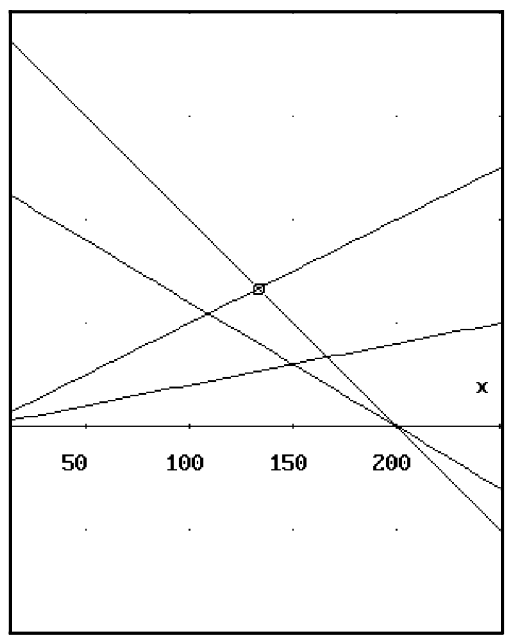

Figure d-8

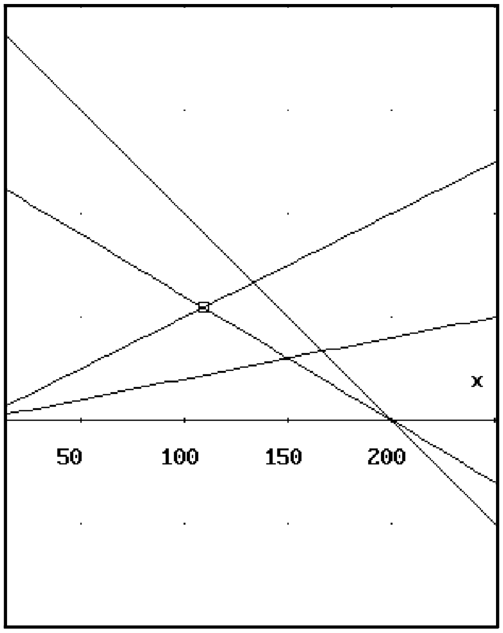

Figure d-9

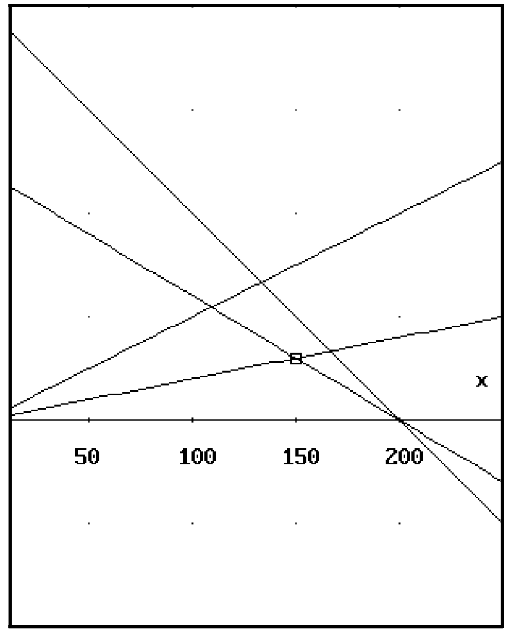

Figure d-10

Now we can determine our exact critical points by solving equations simultaneously

Move to screen #1
<A>uthor [#5,#3] **<ENTER>**
so**<L>**ve expression #8 **<ENTER>**

You should get a=30 and s=150, which is the point of *Figure d-10*

see *Figure d-11*

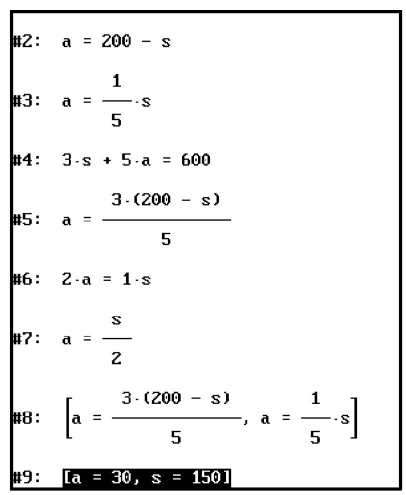

Figure d-11

<**A**>uthor [#2,#3] **<ENTER>**
so<**L**>ve, expression #10 **<ENTER>** appro<**X**> expression #11 **<ENTER>**
You should get a = 33.33 and s = 166.66

see *Figure d-12*

#6: $2 \cdot a = 1 \cdot s$

#7: $a = \frac{s}{2}$

#8: $\left[a = \frac{3 \cdot (200 - s)}{5},\ a = \frac{1}{5} \cdot s\right]$

#9: $[a = 30,\ s = 150]$

#10: $\left[a = 200 - s,\ a = \frac{1}{5} \cdot s\right]$

#11: $\left[a = \frac{100}{3},\ s = \frac{500}{3}\right]$

#12: $[a = 33.3333,\ s = 166.666]$

Figure d-12

<**A**>uthor [#2,#7] **<ENTER>**
so<**L**>ve expression #13 **<ENTER>**
appro<**X**> expression #14 **<ENTER>**

you should get a= 66.66 and s=133.33

see *Figure d-13*

#9: $[a = 30,\ s = 150]$

#10: $\left[a = 200 - s,\ a = \frac{1}{5} \cdot s\right]$

#11: $\left[a = \frac{100}{3},\ s = \frac{500}{3}\right]$

#12: $[a = 33.3333,\ s = 166.666]$

#13: $\left[a = 200 - s,\ a = \frac{s}{2}\right]$

#14: $\left[a = \frac{200}{3},\ s = \frac{400}{3}\right]$

#15: $[a = 66.6666,\ s = 133.333]$

Figure d-13

<**A**>uthor [#5,#7] **<ENTER>**
so<**L**>ve expression #16 **<ENTER>** appro<**X**> expression #17 **<ENTER>**

you should get a=54.54 and s=109.09

see *Figure d-14*

<**Q**>uit

#12: $[a = 33.3333,\ s = 166.666]$

#13: $\left[a = 200 - s,\ a = \frac{s}{2}\right]$

#14: $\left[a = \frac{200}{3},\ s = \frac{400}{3}\right]$

#15: $[a = 66.6666,\ s = 133.333]$

#16: $\left[a = \frac{3 \cdot (200 - s)}{5},\ a = \frac{s}{2}\right]$

#17: $\left[a = \frac{600}{11},\ s = \frac{1200}{11}\right]$

#18: $[a = 54.5454,\ s = 109.090]$

Figure d-14